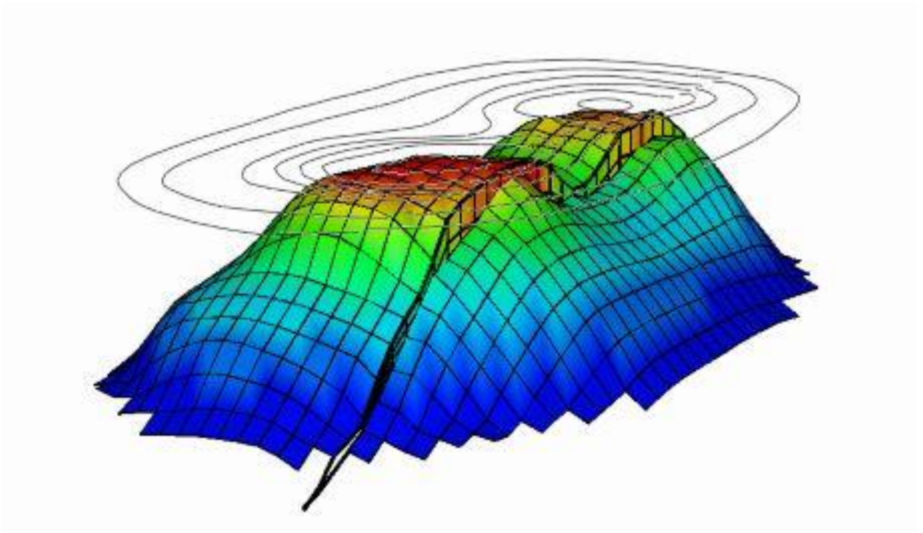
Reservoir Simulation

Author: Ahmed I. Omar



A petroleum reservoir is a porous medium that contains hydrocarbons. The major goal of any reservoir study is to predict future performance of the reservoir and find ways and means of enhancing the ultimate recovery under various operating conditions.

Classical reservoir engineering deals with the reservoir on a gross average basis (tank model) and cannot account adequately for the variations in reservoir and fluid parameters in space and time.

Classical Reservoir Engineering Methods:

- **The analogical methods** utilize features of mature reservoirs that are analogous to the target reservoir in an attempt to forecast the performance of this target zone or reservoir.
- **The experimental methods** measure physical properties, such as pressure, saturation, and/or rates, in laboratory cores and then scale them up to the whole hydrocarbon accumulation.
- **The mathematical methods** use model equations to forecast reservoir performance.

The mathematical methods

Here we briefly mention the mathematical methods because of their direct relationships with reservoir simulation.

1. Material Balance Methods:

The classical material balance methods use a mathematical representation of a reservoir or drainage volume. Their basic principle is based on mass conservation; i.e., the amount of mass for water, oil, or gas remaining in the reservoir after a production period equals the difference of the amount of mass originally in place and that was removed from the reservoir due to production, plus the amount of mass added due to injection and encroachment.

2. Decline Curve Methods:

The classical decline curve methods use one of three mathematical declines (exponential, hyperbolic, and harmonic) to describe the rate of oil production decline.

$$Cq^b = -\frac{1}{q}\frac{dq}{dt},$$

Historical data are matched by choosing the parameters C and b that minimize the error (often using the least squares error) between the data and the equation. Then extrapolating the historical data into the future to predict reservoir performance using the matched equation. A major assumption of any extrapolating method is that all processes occurring in the past will continue in the future. In addition to not giving any spatial or temporal information the decline curve methods cannot be used for “what-if” scenarios.

INTRODUCTION
Classical Methods
Mathematical Methods
Material Balance
MODELLING
APPLICATIONS
TUTORIALS

3. Statistical Methods:

Statistical methods employ empirical correlations that are statistically obtained using the past performance of some reservoirs to forecast the future performance of others. They are a generalization of the analogical methods. A correlation is developed with data from mature reservoirs in the same region, with the same lithology (e.g., carbonate or sandstone) and under the same operating conditions (e.g., solution gas drive or water flood). For the reservoir engineer to be confident in using an empirical correlation model, reservoir properties must be within the limit of the regression database used to develop such a model.

4. Analytical Methods:

Analytical methods, such as pressure-transient and Buckley–Leverett methods, use the analytical solution of a mathematical model. The model consists of a set of differential equations that describe the flow and transport of fluids in a petroleum reservoir, together with an appropriate set of boundary and/or initial conditions. To solve these equations exactly, simplifying assumptions must be made to reduce the complexity of the model. In general, these assumptions are very restrictive. For example, in the pressure-transient method, the assumptions require single-phase flow in a horizontal reservoir with uniform thickness and small pressure differences under the laminar flow condition. The Buckley–Leverett method for two-phase flow ignores gravity and capillary forces under the incompressibility condition. However, since much of the physics of a problem is preserved, the analytical methods are often used to determine how various parameters influence reservoir performance. Furthermore, these methods can be used to validate reservoir simulators.

INTRODUCTION

Classical Methods
Mathematical Methods
Material Balance

MODELLING

APPLICATIONS

TUTORIALS

5. Proxy Models:

The physical model derived for identifying a petroleum reservoir is very complex and contain many unwanted effects of uncertainties and disturbances since the reservoir is dynamic model which continuously change in nonlinear behavior. In recent publications there are some attempts to replace large scale reservoir models with simplified proxy models such as Capacitance Resistive Models. However there are another techniques are implemented lately such as Fully data driven black box models.

Material Balance Methods:

L. P. Dake, The Practice of Reservoir Engineering,

"There should be no competition between material balance and simulation, instead they must be supportive of one another: the former defining the system which is then used as input to the model. Material balance is excellent at history matching production performance but has considerable disadvantages when it comes to prediction, which is the domain of numerical simulation modelling."

Material balance has been used in the industry for the following main purposes:

1. Determining the initial hydrocarbon in place (e.g. STOIIP) by analyzing mean reservoir pressure vs. production data;

2. Calculating water influx i.e. the degree to which a natural aquifer is supporting the production (and hence slowing down the pressure decline);

3. Predicting mean reservoir pressure in the future, **if** a good match of the early pressure decline is achieved and the correct reservoir recovery mechanism has been identified.

4. Predicting the reservoir performance by introducing time-flow rate relationship.

Formation Volume Factor in the Black Oil model

The formation volume factors (FVF) tell how much the oil, gas and water is compressed at a given pressure.

B_o = reservoir volume of oil / standard volume of oil

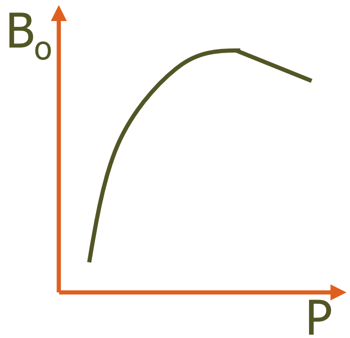
B_g = reservoir volume of gas / standard volume of gas

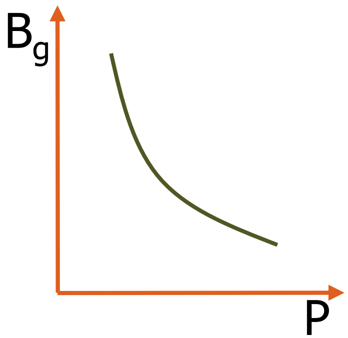
B_w = reservoir volume of water / standard volume of water

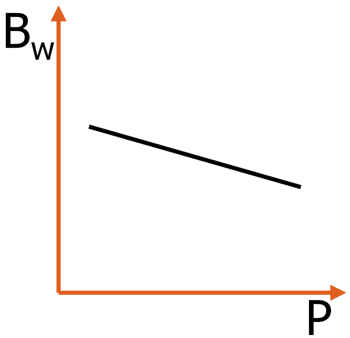
B_o vs. P

B_g vs. P

B_w vs. P







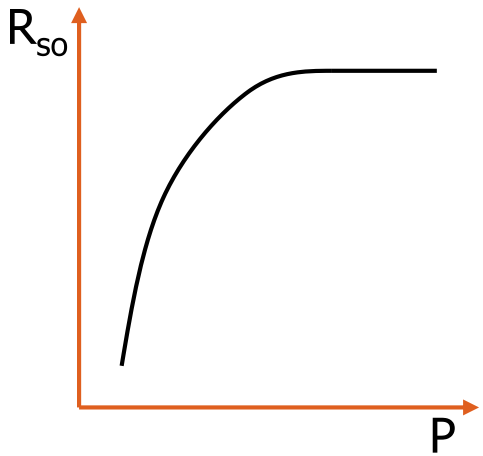
INTRODUCTION
Classical Methods
Mathematical Methods
Material Balance
MODELLING
APPLICATIONS
TUTORIALS

Solution Gas-Oil Ratio in the Black Oil model

The Rso plot shows how the solution gas ratio develops vs pressure. When the pressure reaches the bubble point pressure, it is no longer possible to solve more gas into the oil. Thus the gradient of the curve becomes zero.

R_s = standard volume gas / standard volume oil

R_{so} vs. P



Material Balance Methods:

Symbols		Units	Units SI
B_g	Gas formation volume factor	bbl/SCF	M^3/SCM
B_o	Oil formation volume factor	bbl/STB	M^3/SCM
B_t	Total formation volume factor	bbl/STB	M^3/SCM
B_w	Water formation volume factor	bbl/STB	M^3/SCM
c_f	Pore compressibility	vol/vol/psi	vol/vol/Mpa
c_w	Water compressibility	vol/vol/psi	vol/vol/Mpa
G	Initial gas-cap volume	SCF	SCM
G_p	Cumulative gas produced = $G_{ps} + G_{pc}$	SCF	SCM
G_{ps}	Cumulative solution gas produced	SCF	SCM
G_{pc}	Cumulative gas cap produced	SCF	SCM
m	Ratio initial reservoir free gas volume to initial reservoir oil volume	bbl/bbl	M^3/M^3

INTRODUCTION
Classical Methods
Mathematical Methods
Material Balance
MODELLING
APPLICATIONS
TUTORIALS

Material Balance Methods:

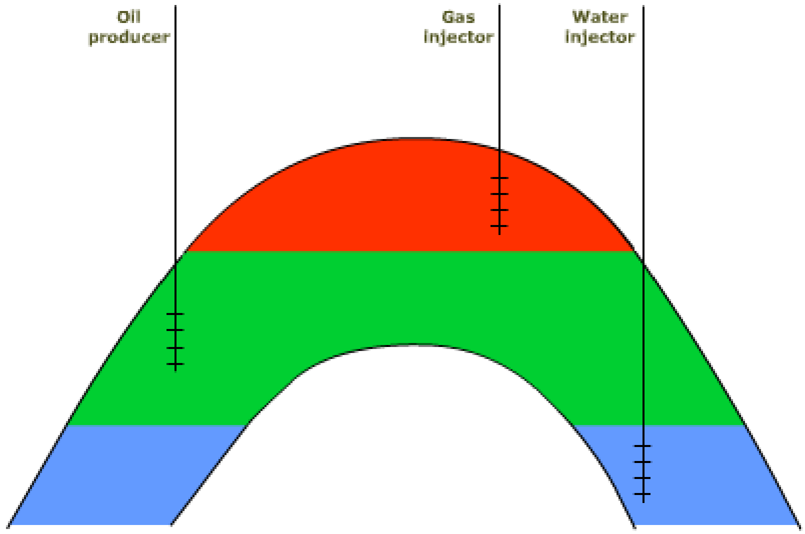
N	Stock tank oil initially in place	STB	STM^3
N_p	Cumulative tank oil produced	STB	STM^3
p	Average reservoir pressure	psi	MPa
p_i	Initial reservoir pressure	psi	MPa
R_p	Cumulative gas/oil ratio	SCF/STB	SCM/STM^3
R_s	Solution gas/oil ratio	SCF/STB	SCM/STM^3
S_w	Average connate water saturation	fraction	fraction
W_e	Cumulative water influx	bbl or STB	M^3 or STM^3
W_p	Cumulative water production	bbl or STB	M^3 or STM^3
	Other subscripts		
i	at initial conditions		
b	at bubble point		

Material Balance Methods:

$$\left\{\begin{array}{c}\text{Amount of fluids present}\\ \text{in the reservoir initially}\\ \text{(st. vol.)}\end{array}\right\} - \left\{\begin{array}{c}\text{Amount of}\\ \text{fluids produced}\\ \text{(st. vol.)}\end{array}\right\} = \left\{\begin{array}{c}\text{Amount of fluids remaining}\\ \text{in the reservoir finally}\\ \text{(st. vol.)}\end{array}\right\}$$

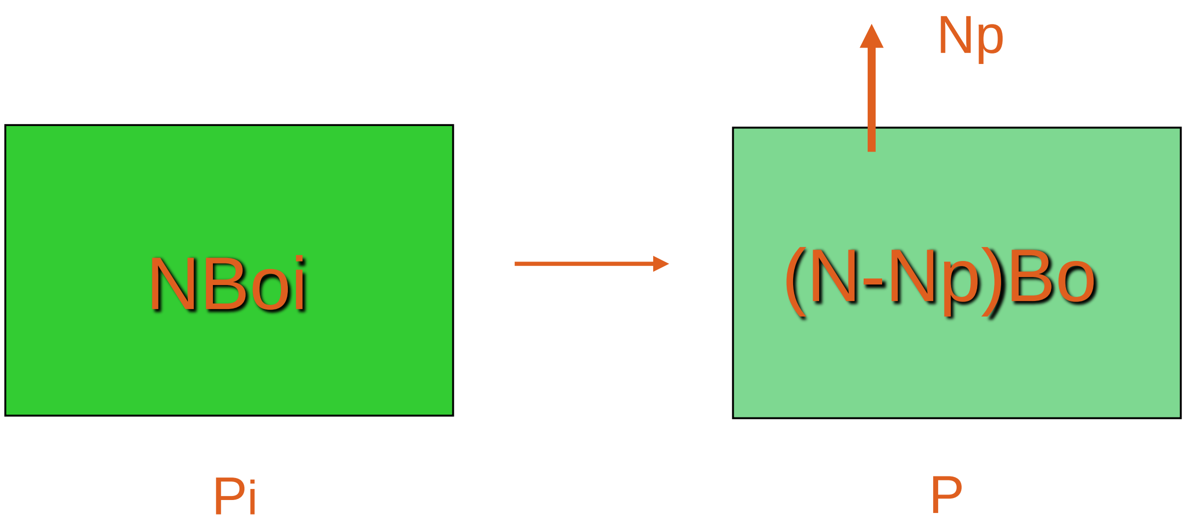
Note that “fluids produced” include all influence on the reservoir:

- Production
- Injection
- Aquifer influx



Material Balance Methods:

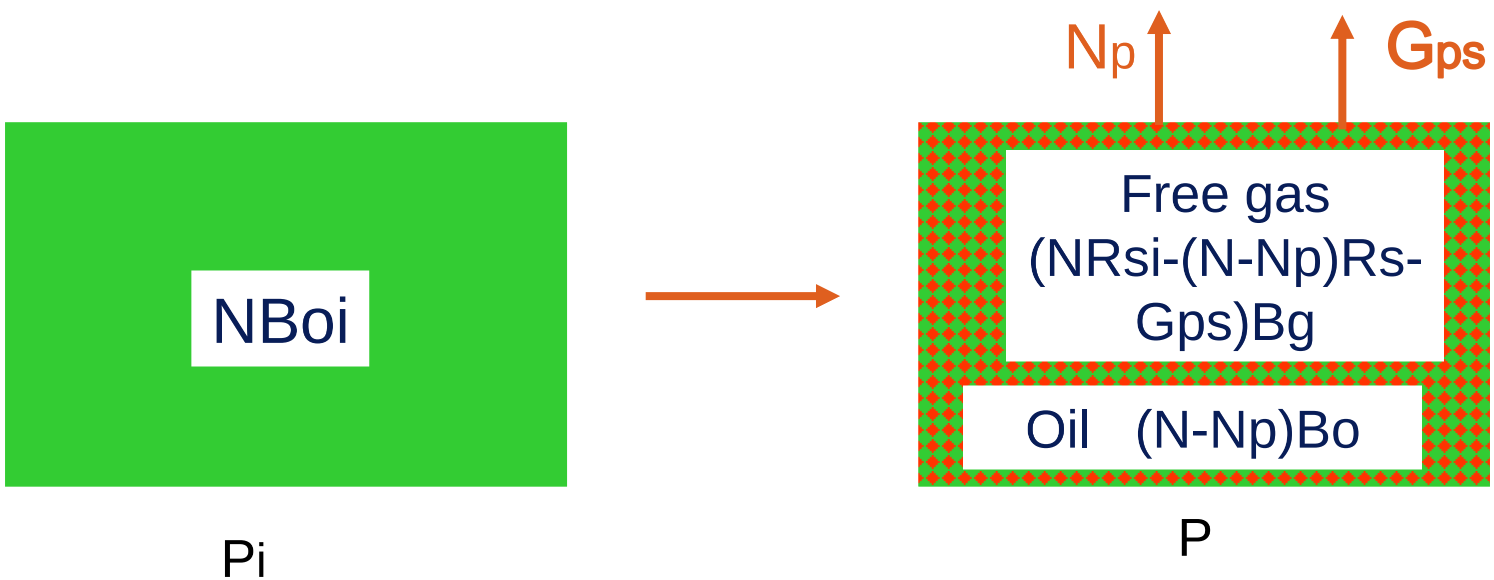
MB for Oil Reservoirs Above Bubble Point:



$$NB_{oi} = \left(N - N_p\right)B_o$$

Material Balance Methods:

MB for Oil Reservoirs Below Bubble Point:

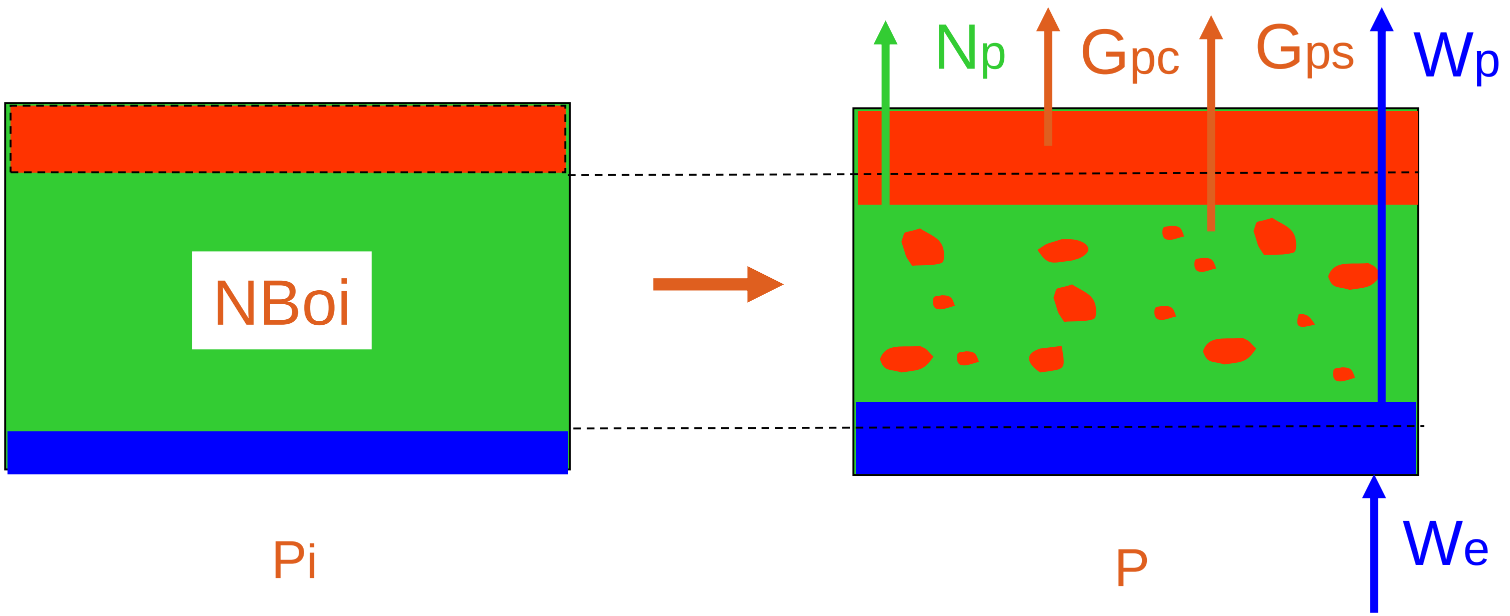


$$\boxed{N = \frac{N_p B_o + \left(G_{ps} - N_p R_s\right) B_g}{B_o - B_{oi} + \left(R_{si} - R_s\right) B_g}}$$

INTRODUCTION
Classical Methods
Mathematical Methods
Material Balance
MODELLING
APPLICATIONS
TUTORIALS

Material Balance Methods:

MB with gas cap and water drive:



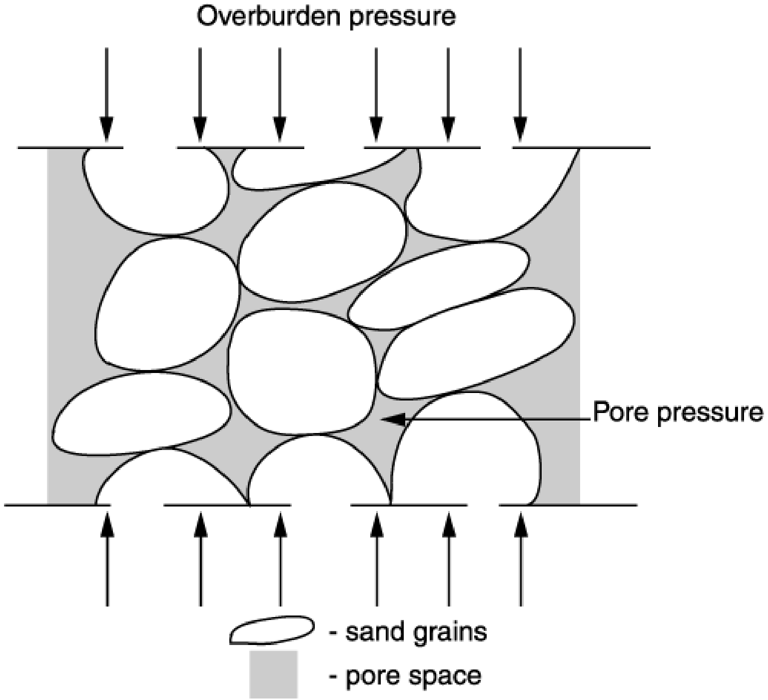
$$\left(W_e - W_p\right) + \left(G - G_{pc}\right)B_g - GB_{gi}$$
$$= NB_{oi} - \left(\left(N - N_p\right)B_o + \left(NR_{si} - \left(N - N_p\right)R_s - G_{ps}\right)B_g\right)$$

INTRODUCTION
Classical Methods
Mathematical Methods
Material Balance
MODELLING
APPLICATIONS
TUTORIALS

Material Balance Methods:

Effect of Pore Volume Changes:

1. Impact of pore volume changes due to rock:



$$\boxed{c_f = \frac{1}{V_p}\frac{\Delta V_{pr}}{\Delta p}}$$

$$\boxed{\Delta V_{pr} = c_f \Delta p V_p}$$

Material Balance Methods:

Effect of Pore Volume Changes:

1. Impact of pore volume changes due to connate water:

Compressibility of water:

$$\boxed{c_w = \frac{1}{V_{pw}} \frac{\Delta V_{pw}}{\Delta p}}$$

$$\boxed{\Delta V_{pw} = c_w \Delta p S_{wc} V_p}$$

Total Pore Volume Change due to rock & water:

$$\boxed{\Delta V_p = \Delta V_{pr} + \Delta V_{pw}}$$

$$\boxed{\therefore \Delta V_p = \left(c_f + c_w S_{wc}\right) \Delta p V_p}$$

Material Balance Methods:

General Material Balance Equation

Net water influx + gas cap expansion + pore volume reduction =

$$\overline{\left(W_e - W_p B_w\right)} + \overline{\left(G - G_{pc}\right)B_g - GB_{gi}} + \overline{\frac{\left(c_f + c_w S_{wc}\right)}{\left(1 - S_{wc}\right)}\Delta p N B_{oi}}$$

$$= \underline{NB}_{oi} - \left(\underline{\left(N - N_p\right)B_o + \left(NR_{si} - \left(N - N_p\right)R_s - G_{ps}\right)B_g}\right)$$

Original oil volume – volume of remaining oil and free solution gas.

INTRODUCTION
Classical Methods
Mathematical Methods
Material Balance
MODELLING
APPLICATIONS
TUTORIALS

Material Balance Methods:

Predicting the reservoir performance using Material Balance Equation:

✓ Material balance in the form of straight line :

Material balance not a difficult concept but it is difficult in applying it to real reservoirs. There is often inadequate understanding of drive mechanisms.

Odeh & Havlena (1963) rearranged MB equation into different linear forms. Their method requires the plotting of a variable group against another variable group selected depending on the drive mechanism.

The technique is referred to as history matching.

Material Balance Methods:

Predicting the reservoir performance using Material Balance Equation:

✓ **Material balance in the form of straight line :**

Material balance can be rewritten as:

$$N_p\left[B_o+\left(R_p-R_s\right)B_g\right]+W_pB_w-W_{inj}$$

$$=N\left[\left(B_o-B_{oi}\right)+\left(R_{si}-R_s\right)B_g\right]$$

$$+mNB_{oi}\left(\frac{B_g}{B_{gi}}-1\right)+\frac{(1+m)NB_{oi}\left(c_wS_s+c_f\right)\Delta p}{\left(1-S_{wc}\right)}+W_e$$

INTRODUCTION
Classical Methods
Mathematical Methods
Material Balance
MODELLING
APPLICATIONS
TUTORIALS

Material Balance Methods:

Predicting the reservoir performance using Material Balance Equation:

✓ **Material balance in the form of straight line :**

Havlena and Odeh simplified equation to:-

$$F = NE_o + NmE_g + NE_{fw} + W_e$$

$$F = N_p\left[B_o + \left(R_p - R_s\right)B_g\right]$$

$$E_o = \left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g \text{....bbl/STB}$$

$$E_{fw} = \frac{(1+m)B_{oi}\left(c_w S_{wc} + c_f\right)\Delta p}{\left(1 - S_{wc}\right)} \text{.....} bbl / STB$$

$$E_g = mNB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) \text{...} bbl / STB$$

INTRODUCTION
Classical Methods
Mathematical Methods
Material Balance
MODELLING
APPLICATIONS
TUTORIALS

Material Balance Methods:

Predicting the reservoir performance using Material Balance Equation:

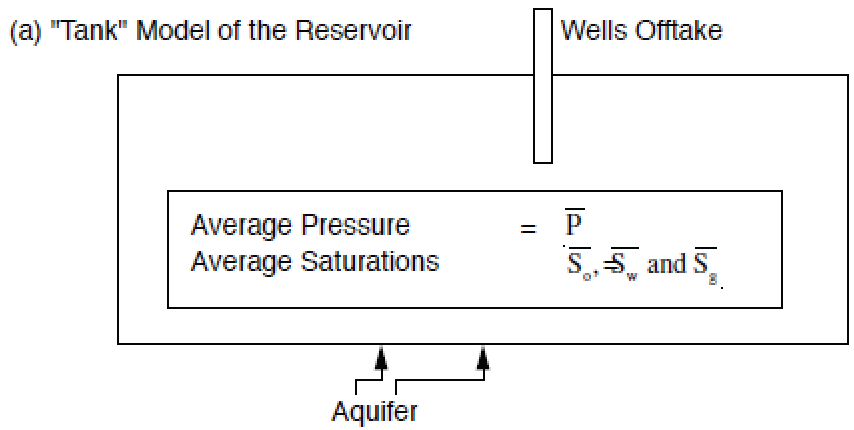
✓ **Material balance in the form of straight line :**

Tarek Ahmed, Reservoir Engineering Handbook, Third edition, Chapter 11

1. Case 1: Determination of N in volumetric undersaturated reservoirs
2. Case 2: Determination of N in volumetric saturated reservoirs
3. Case 3: Determination of N and m in gas cap drive reservoirs
4. Case 4: Determination of N and We" in water drive reservoirs
5. Case 5: Determination of N, m, and We in combination drive reservoirs
6. Case 6: Determination of average reservoir pressure, p

Material Balance Methods:

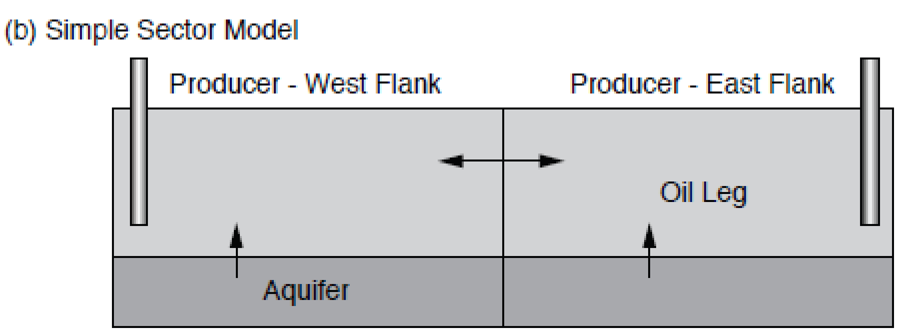
The particular advantage of material balance models is that they are very simple. They can address questions relating to average field pressure for given quantities of oil/water/gas production and water influx from given initial quantities and initial pressure (within certain assumptions).



Material Balance Methods:

However, the material balance model is essentially a ***tank*** model, it cannot address questions about why the pressures in two sectors of the reservoir are different (since a single average pressure in the system is a core assumption).

The ***sector model*** is somewhat more complex in that it recognizes different regions of the reservoir.



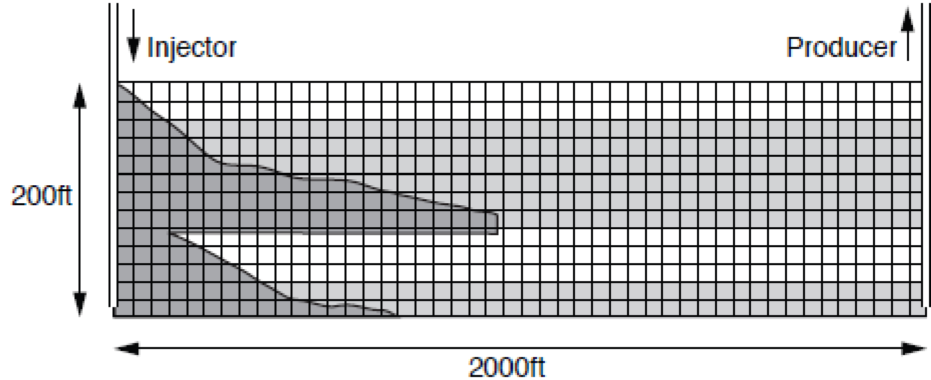
Material Balance Methods:

Material balance can not answer the more detailed questions especially for mature field like:

where should I locate an infill well and should it be vertical, slanted or horizontal ?

For such complicated questions, the model has to be more detailed and it contains more spatial information. Like **Simulation Model**

(c) Fine Grid Simulation Model of a Waterflood



INTRODUCTION
MODELLING
APPLICATIONS
TUTORIALS

What is a simulation model?

A **simulation model** shows the main features of a real system or resembles it in its behavior , but is simple enough to make calculations on. These calculations may be "analytical" or "numerical". By analytical it means the equations that represent the model can be solved using mathematical techniques such as those used to solve algebraic or differential equations. An analytic solution would normally be written in terms of “well known” equations or functions (x2, sin x, ex etc)..

Learning goals

- Basic understanding of material balance